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Coplanar Elliptical Orbit Transfer Using Aerocruise

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The problem of coplanar symmetrical elliptical terrestrial orbit transfer is studied using the aerocruise technique. To facilitate the solution, it is assumed that during the atmospheric flight the thrust will cancel the aerodynamic drag. An optimal control problem is formulated to obtain the lift controls that minimize the fuel consumption. The complete two-point boundary value problem is solved numerically, and results are compared with aeroassisted and pure propulsive modes. The aerocruise maneuver is shown to be most advantageous, provided perigee of the symmetric orbits is not too much higher than the atmospheric radius. Further investigation is needed to calculate heating rates for the vehicles with different configurations and the effect of higher maximum lift-to-drag ratio on fuel consumption in detail.

Nomenclature

B = $\rho_0 S H_e C_L^*/2m_0$
b = R/H_e
C_L = lift coefficient
C_L^* = value of C_L at $(L/D)_{max}$
C_T = $I_{sp}\surd(\mu/R^3)$
D = drag force
E^* = $(L/D)_{max}$
e = eccentricity of the trajectory
H_s = altitude of vehicle
H = Hamiltonian
H_e = entry altitude, 120 km
L = lift force
m = vehicle mass
$\boldsymbol{p}$ = adjoint vector
p_j = adjoint variable associated with state j
R = radius of sensible atmosphere of the Earth
r = radial distance from center of the Earth
r_a = apogee distance
r_p = perigee distance
r_r = reference altitude $+r_0$
r_0 = radius of the Earth
S = surface area
T = thrust magnitude
t = time
V = velocity of spacecraft
v = $V/\surd(\mu/R)$
β = inverse of atmosphere height scale
γ = flight-path angle
Δ = rotation angle of apsidal line
ΔV = thrust impulse
δ = ρ/ρ_0
θ = rotation angle during atmospheric flight
λ = normalized lift control, C_L/C_L^*
μ = gravitation constant multiplied by mass of the Earth (398601.3 km^3/s^2); mass ratio m/m_0
ρ = density
ρ_0 = value of ρ at reference altitude

Subscripts

e = value at atmospheric entry
f = value at atmospheric exit; final condition
I = single impulse
0 = initial value at atmospheric entry

Introduction

IN 1962,[1] London published a paper in which he presented the novel idea of using aeroassisted maneuvering to change planetary orbits. Since then numerous investigations have been conducted in the area of aeroassisted orbit transfer for terrestrial and extraterrestrial missions. In 1993, the Magellan spacecraft changed the idea into reality by altering its Venus orbit using aerobraking techniques, thereby consuming far less fuel compared with a pure-propulsive orbital change. Recently, a technique known as aerogravity assist has been proposed to augment gravity assist by a planetary atmospheric maneuver.[2,3]

Vinh and Johannesen[4] have also considered aeroassisted maneuvering to change the apsidal line of coplanar elliptical orbits and compared pure-propulsive with aeroassisted techniques. However, their analysis was restricted to aeroglide mode only (which is, in fact, aerobraking mode). The aerocruise mode, in which propulsive and aerodynamic forces are used simultaneously, is investigated in several studies for orbital plane changes.[5,6] The problem of minimum fuel transfer between coplanar elliptical orbits using aerocruise is addressed for the first time in the present paper. The orbital transfer is carried out in such a way that only the apsidal line of the Earth orbit is rotated.

The basic events of orbit transfer, in our case, are as follows. The spacecraft is launched from an initial elliptical orbit by applying a thrust impulse opposite to the velocity of the spacecraft so as to inject it into an elliptical orbit with perigee inside the atmosphere. During maneuvering through the atmosphere, the aerodynamic lift is used for maneuvering, and the aerodynamic drag is canceled by the thrust. The angle of rotation during the atmospheric flight is used to change the Earth orbit.

The optimization of the atmospheric trajectory is carried out using the Hamiltonian formulation based on Pontryagin's maximum principle in such a way that minimum fuel is consumed during the atmospheric flight. A comparison of characteristic velocities (ΔV) for aerocruise, aeroassist, and all propulsive maneuvers is given for different rotation angles of the apsidal line along with the nature of the atmospheric trajectory.

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Equations of Motion

The equations of motion for planar thrusting and lifting flight are[7]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = V\sin\gamma, \qquad \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{V\cos\gamma}{r}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{T-D}{m} - \frac{\mu}{r^2}\sin\gamma \tag{1}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{L}{mV} + \left(\frac{V}{r} - \frac{\mu}{r^2 V}\right)\cos\gamma, \qquad \frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{T}{\mu/r^2}\frac{1}{I_{\mathrm{sp}}}$$

To facilitate the solution, it is assumed that aerodynamic drag is canceled with thrust. The equations of motion using a parabolic drag polar (for a discussion of parabolic drag polar the interested reader is referred to Ref. 7) in nondimensionalized form with θ as the independent variable are given by

$$\frac{\mathrm{d}h}{\mathrm{d}\theta} = (b-1+h)\tan\gamma, \qquad \frac{\mathrm{d}v}{\mathrm{d}\tau} = -\frac{b\tan\gamma}{(b-1+h)v}$$

$$\frac{d\gamma}{\mathrm{d}\theta} = \frac{\delta B(b-1+h)\lambda}{\mu\cos\gamma} - \frac{b}{v^2(b-1+h)} + 1 \tag{2}$$

$$\frac{d\mu}{\mathrm{d}\theta} = -\frac{\delta B(1+\lambda^2)(b-1+h)^3 v}{2E^* b^2 C_T\cos\gamma}$$

Optimization Problem

The obvious performance index is the final fuel mass, which is to be maximized (or the characteristic velocity ΔV is minimized) and, thereby, propellant consumption is minimized. The fuel mass saving is directly proportional to the cost saving. The total characteristic velocity

$$\Delta V = \Delta V_1 + 2\Delta V_2 \tag{3}$$

where ΔV_1 is the loss of the velocity of the spacecraft in terms of fuel consumption during atmospheric flight due to aerodynamic drag and ΔV_2 is the tangential thrust applied at points A and B in Fig. 1.

Now we proceed to derive the necessary optimal conditions using Pontryagin's maximum principle. The adjoint variables are denoted by p_h, p_v, p_γ, and p_μ. We have the Hamiltonian

$$H = p_h(b-1+h)\tan\gamma + p_v\left[-\frac{b\tan\gamma}{(b-1+h)v}\right] + p_\gamma\left[\frac{\delta B(b-1+h)\lambda}{\mu\cos\gamma} - \frac{b}{v^2(b-1+h)} + 1\right] + p_\mu\left[-\frac{\delta B(1+\lambda^2)(b-1+h)^3 v}{2E^* b^2 C_T\cos\gamma}\right] \tag{4}$$

Using the first optimal condition with respect to the normalized lift control λ, H is maximized when $\mathrm{d}H/\mathrm{d}\lambda = 0$, which leads to

$$\lambda = \frac{p_\gamma}{p_\mu}\frac{E^* C_T b^2}{\mu(b-1+h)^2} \tag{5}$$

Here we examine the second derivative of H with respect to λ, which is negative satisfying the optimal condition of Pontryagin's maximum principle. By introducing new parameters $x = p_h/p_\mu$ and $y = p_v/p_\mu$, differentiating λ, x, and y with respect to θ, and using Eq. (4), we have the modified adjoint equations

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\theta} = \frac{E^* C_T b^3 y}{\mu(b-1+h)^3 v^2\cos^2\gamma} - \frac{E^* C_T b^2 x}{\mu(b-1+h)^3 v\cos^2\gamma} + \frac{\delta B(b-1+h)(1-\lambda^2)\sin\gamma}{2\mu\cos^2\gamma} - \frac{\delta B(b-1+h)^3 v\lambda^3}{\mu E^* b^2 C_T\cos\gamma}$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{\delta B(b-1+h)^2 v[(3+\lambda^2)]}{2E^* b^2 C_T\cos^2\gamma} - \frac{\lambda\mu}{E^* b v C_T} - x\tan\gamma - \frac{b\tan\gamma y}{(b-1+h)^2 v} - \frac{\delta B(b-1+h)^2 v\lambda^2 x}{\mu E^* b^2 C_T\cos\gamma} - \frac{\dot{\delta} B(b-1+h)^3(1-\lambda^2)v}{2E^* b^2 C_T\cos\gamma} \tag{6}$$

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -\frac{b\tan\gamma y}{(b-1+h)v^2} - \frac{2\mu(b-1+h)\lambda}{E^* b v^2 C_T} + \frac{\delta B(b-1+h)^3(1+\lambda^2)}{2E^* b^2 C_T\cos\gamma} - \frac{\delta B(b-1+h)^3 v\lambda^2 y}{\mu E^* b^2 C_T\cos\gamma}$$

where

$$\dot{\delta} = \frac{\mathrm{d}\delta}{\mathrm{d}h} = \frac{H_e}{\rho_0}\frac{\mathrm{d}\rho}{\mathrm{d}H} = -\beta H_e e^{-\beta(r-r_r)}$$

The modified Hamiltonian $\hat{H}$, a function of x,y, and λ, is given by

$$\hat{H} = x(b-1+h)\tan\gamma + y\left[-\frac{b\tan\gamma}{(b-1+h)v}\right] + \frac{\lambda\mu(b-1+h)^2}{E^* b^2 C_T\cos\gamma}\left[\frac{\delta B(b-1+h)\lambda}{\mu\cos\gamma} - \frac{b}{v^2(b-1+h)} + 1\right] - \left[\frac{\delta B(1+\lambda^2)(b-1+h)^3 v}{2E^* b^2 C_T\cos\gamma}\right] \tag{7}$$

and the transversality conditions ($\lambda_f = 0$ and $y_f = 0$ at $h_f = 1.0$).

Now we have seven equations; four for the states and three for the adjoints λ, x, and y. Integration of these will yield extremal trajectories for a number of situations, which differ only in the entry and exit conditions. We have a two-point boundary value problem with seven boundary conditions to be satisfied. Initial values of h, v, γ, and μ must be specified at $\theta = \theta_e$, and final values of λ_f, y_f, and

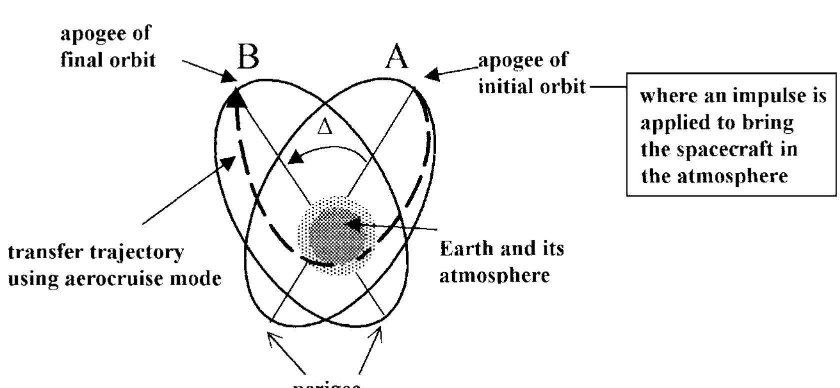

Fig. 1 Symmetrical elliptical orbit transfer using aerocruise.

h_f are specified at $\theta = \theta_f$. Hence, the problem is a three-parameter problem and $\hat{H} \neq 0$ because θ_f is specified. The initial values of λ, x, and y are to be guessed during the course of solving the two-point boundary value problem.

Method of Solution

The optimization problem was solved for three angles of rotation of the apsidal lines of the elliptical orbits, namely, 10, 20, and 30 deg. The two-point boundary value problem was solved by the shooting method. Using the computed values of v_e and γ_e, setting $h_e = 1$, and choosing initial guesses for λ_e, x_e, and y_e as initial conditions, Eqs. (2) and (6) are integrated until h_f becomes 1 again and the transversality conditions are satisfied. The integration is performed by a Fehlberg fourth- fifth-order Runge–Kutta method with local error controlled to be less than $1.0 \times 10^{-6}\%$. The necessary optimal conditions are derived using Maple V. In all cases studied, it has been possible to find a set of initial values of λ, x, and y such that the boundary conditions are satisfied. The trajectory and control are optimal because the necessary conditions are satisfied.

Results and Discussion

For system parameters, we choose the value $B = 0.046$, which corresponds to an altitude of 75 km for a typical vehicle for which the aerodynamic vehicle characteristic value $E^* = 1.5$, and $C_T = 0.3$ for the propulsion system, which corresponds to a specific impulse value of about 249 s. The radius of sensible atmosphere R is taken as 6498 km, i.e., an altitude of 120 km, inasmuch as above this altitude the atmospheric density was assumed to be identically zero. Over the altitude of atmospheric flight, 75–120 km, the atmospheric density is approximated by using an exponential model. The apogee distance r_a for all the cases is selected as 12,000 km.

The results of the analysis are presented in two parts. In the first part, we discuss the optimal atmospheric trajectories. In the second part, we compare characteristic velocities associated with orbital transfer using all-propulsive, aeroassisted, and aerocruise modes.

Nature of the Atmospheric Trajectory

Figure 2 shows the variation of altitude, V, γ, μ, and λ with θ for the optimal atmospheric trajectory for an angle of rotation $\Delta = 30$ deg and $r_p = 7000$ km. Figure 2 reveals that the vehicle enters in the atmosphere with negative flight-path angle γ_e, and during most of the atmospheric flight it flies with $L/D \approx E^*$, i.e., $\lambda \approx -1$, to limit fuel consumption. The exit flight-path angle is slightly less than γ_e, which will not much affect the final orbit. The initial velocity V_e and exit velocity V_f are equal, and the variation in velocity during atmospheric flight is due to the change in altitude. Table 1 shows a consistent increase in fuel consumption with the increase in rotation angle Δ. It was also found that for a given value of B the maximum convective heating rates for the all of the cases studied were around 15 W/cm^2.

The atmospheric trajectory of the spacecraft for an actual mission would be similar to the one computed in this study. At the time of entry in the atmosphere, the spacecraft would fly with negative flight-path angle until it reached the altitude where flight with $\lambda \approx -1$ is possible. After maneuvering with $\lambda \approx -1$ for the desired angle of rotation, the spacecraft would exit the atmosphere. Because the flight would be thrusting, it would possibly be more stable than that of a nonthrusting one, such as that of the aerogravity assist.

Table 1 Final mass ratio of fuel μ_f for different rotational angles for the AC mode

Rotational angle Δ, deg	Final mass ratio μ_f
10	0.765
20	0.690
30	0.621

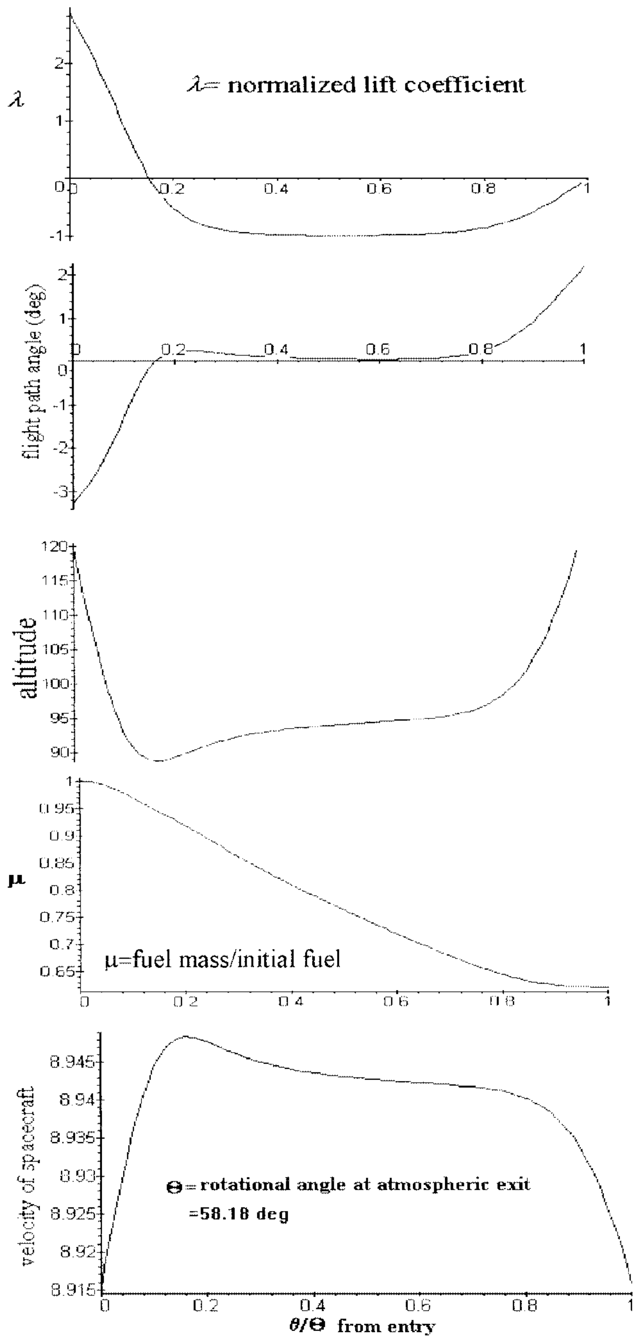

Fig. 2 Time history of state variables and normalized lift coefficient.

Comparison of Aerocruise with Aeroassist and Pure-Propulsive Techniques

We have taken three values of Δ, namely, 10, 20, and 30 deg, and Table 2 shows the corresponding results for aerocruise (AC), aeroassist (AE), i.e., the aerobraking technique (independent of angle of rotation of apsidal line Δ), and the pure-propulsive technique with single impulse (the data are not available for optimal double impulse for the 10-, 20-, and 30-deg values of Δ). The results show that for the data given earlier, AC is more advantageous than AE and pure-propulsive single impulse ΔV_{I}. From Ref. 4, we take data for coplanar elliptical orbit transfers and compare the characteristic velocities for a pure-propulsive double impulse (ΔV_{II}) and aeroassisted technique, i.e., aerobraking technique (ΔV_{AE}), with that of AC (ΔV_{AC}). For this case, $r_p = R$, $\Delta = 80$ deg, and $e = 0.5$, and the relevant characteristic velocities are $\Delta V_{\mathrm{II}} = 1.45$ km/s, $\Delta V_{\mathrm{AE}} = 1.44$ km/s, and ΔV_{AC} for $E^* = 5$ calculated as 1.04 km/s. It shows that even for large Δ, the AC technique saves fuel, provided E^* is high and r_p is not much higher than R (as it is shown in Table 2). The mass ratio μ_f is approximately 0.65 for $E^* = 5$, which is directly related to E^*, because for $E^* = 1.5$, $\mu_f = 0.25$. Hence, if the value

Table 2 Comparison of characteristic velocity

R_p, km	Eccentricity e	Characteristic velocity, 10 deg ΔV_{I}	10 deg ΔV_{AC}	20 deg ΔV_{I}	20 deg ΔV_{AC}	30 deg ΔV_{I}	30 deg ΔV_{AC}	$\leq$180 deg ΔV_{AE}
70000	0.2631	0.8639	0.521	1.730	0.621	2.592	0.747	1.485
8000	0.2000	0.8996	0.951	1.798	1.032	2.689	1.173	1.736
9000	0.1428	0.9308	1.311	1.858	1.392	2.775	1.532	2.231

of E^* is increased further, then μ_f would also be increased. Furthermore, it should be noted that in the case of the aeroassisted technique, which is actually the aerobraking technique, the time required for orbit transfer would be very long, as was observed in the case of the Magellan mission.

Conclusion

The minimum fuel aerocruise coplanar elliptical symmetrical orbit transfer has been considered. The necessary conditions were derived for the optimization problem, and the optimal atmospheric trajectory was determined numerically. It was found that the spacecraft flies most of the time with L/D close to E^*, i.e., $\lambda = -1$, during the atmospheric trajectory. It gives confidence in our results because it is one of the necessary conditions for the optimal flight to limit fuel consumption. The results show that the AC technique saves fuel even for a large rotation angle of apsidal line, provided E^* is high and r_p is not much higher than the atmospheric radius. Further research is needed to study in detail the fuel savings in the case when the vehicle with high E^*, such as $E^* = 5$ or more, and a different configuration is considered.

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